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Kepler E-Banhatti and Modified Kepler E-Banhatti Indices

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Abstract

We introduce the Kepler E-Banhatti index, the modified Kepler E-Banhatti index and their corresponding exponentials of a graph. Furthermore, we compute these newly defined Kepler E-Banhatti indices for friendship graphs, wheel graphs and certain networks like chain silicate networks. Also we establish some properties of the Kepler E-Banhatti index.

Keywords: Kepler E-Banhatti index, modified Kepler E-Banhatti index, chain silicate networks.

2020 MSC: 05C07, 05C09, 05C92.

1. Introduction

Let G be a finite, simple, connected graph. Let V(G) be the vertex set and E(G) be the edge set of G. The degree d(u) of a vertex u is the number of vertices adjacent to u. The edge e connecting the vertices u and v is denoted by uv. If e=uv is an edge of G, then the vertex u and edge e are incident as are v and e. Let d(e) denote the degree of an edge e=uv and defined as d(e)=d(u)+d(v)-2. For undefined terms and notations, we refer [16, 18]. A graph index is a numerical parameter mathematically derived from the graph structure. The graph indices have their applications in various disciplines of Science and Technology [1, 2, 3, 4, 5] .

The Banhatti degree of a vertex u in a graph G defined as [20],

$$B(u) = \frac{d(e)}{n - d(u)},$$

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where $|V(G)| = n$ and the vertex u and edge e are incident in G. The first and second E-Banhatti indices and their polynomials were defined by Kulli in [20] as,

$$EB_1(G) = \sum_{uv \in E(G)} [B(u) + B(v)],$$

$$EB_2(G) = \sum_{uv \in E(G)} B(u) \times B(v).$$

Kulli introduced the product connectivity E-Banhatti index and the reciprocal product connectivity E-Banhatti index of a graph G and they are defined as [21] as, The product connectivity E-Banhatti index,reciprocal product connectivity E-Banhatti index of a graph G and their polynomials defined by kulli in [21] as,

$$PEB(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{B(u) \times B(v)}},$$

$$RPEB(G) = \sum_{uv \in E(G)} \sqrt{B(u) \times B(v)}.$$

The E-Banhatti Sombor index of a graph G defined in [17] as,

$$EBS(G) = \sum_{uv \in E(G)} \sqrt{B(u)^2 + B(v)^2}.$$

The Kepler Banhatti index was introduced by Kulli in [19] and it is defined as,

$$KB(G) = \sum_{uv \in E(G)} [d(u) + d(v) + \sqrt{d(u)^2 + d(v)^2}].$$

Motivated by the definition of Kepler Banhatti index [11, 12, 13, 14, 15], we introduce the Kepler E-Banhatti index of a graph and it is defined as

$$KEB(G) = \sum_{uv \in E(G)} [B(u) + B(v) + \sqrt{B(u)^2 + B(v)^2}].$$

Considering the Kepler E-Banhatti index, we introduce the Kepler E-Banhatti exponential of a graph G and defined it as

$$KEB(G, x) = \sum_{uv \in E(G)} x^{\left[B(u)+B(v)+\sqrt{B(u)^2+B(v)^2}\right]}.$$

We define the modified Kepler E-Banhatti index of a graph G as

$$m_{KEB(G)} = \sum_{uv \in E(G)} \left[\frac{1}{B(u) + B(v) + \sqrt{B(u)^2 + B(v)^2}}\right].$$

Considering the modified Kepler E-Banhatti index, we introduce the modified Kepler E-Banhatti exponential of a graph G and defined it as

$$m_{KEB(G,x)} = \sum_{uv \in E(G)} x^{\left[\frac{1}{B(u)+B(v)+\sqrt{B(u)^2+B(v)^2}}\right]}.$$

Several graph indices have been defined so far and they have applications in many areas such as , pharmacology, toxicology,environmental chemistry and theoretical chemistry [6, 7, 8, 9, 10] .

2. Mathematical Properties

Theorem 2.1. Let G be a graph. Then

$$KEB(G) \geqslant (1+\frac{1}{\sqrt{2}})EB_1(G),$$

with equality if only if G is regular.

Proof. , By the Jensen inequality, for a concave function f(x),

$$f\left(\frac{1}{n}\sum x_i\right) \geqslant \frac{1}{n}\sum f\left(x_i\right),$$

with equality for a strict concave function if and only if $x_1 = x_2 = ... = x_n$.Choosing $f(x) = \sqrt{x}$, We obtain

$$\sqrt{\frac{B(u)^2 + B(v)^2}{2}} \geqslant \frac{B(u) + B(v)}{2}.$$

Thus

$$(B(u) + B(v)) + \sqrt{B(u)^2 + B(v)^2} \geqslant (B(u) + B(v)) + \frac{1}{\sqrt{2}}(B(u) + B(v)).$$

Hence

$$\sum_{uv\in E(G)} [(B(u) + B(v)) + \sqrt{B(u)^2 + B(v)^2}] \geqslant (1+\frac{1}{\sqrt{2}}) \sum_{uv\in E(G)} (B(u) + B(v)).$$

Thus

$$KEB(G) \geqslant (1+\frac{1}{\sqrt{2}})EB_1(G),$$

□

with equality if and only if G is regular.

Theorem 2.2. Let G be a graph. Then

$$KEB(G) \leqslant (1+\frac{1}{\sqrt{2}})EB_1(G) - \sqrt{2}RPEB(G).$$

Proof. It is known that for $1 \leqslant x \leqslant y$

$$f(x, y) = (x + y - \sqrt{xy}) - \sqrt{\frac{x^2 + y^2}{2}}.$$

is decreasing for each y.
Thus

$$f(x, y) \geqslant f(y, y) = 0.$$

Hence

$$x+y-\sqrt{xy} \geqslant \sqrt{\frac{x^2+y^2}{2}}$$

or

$$\sqrt{\frac{x^2+y^2}{2}} \leqslant x+y-\sqrt{xy}$$

put $x = B(u)$ and $y = B(v)$, we get

$$\sqrt{\frac{B(u)^2+B(v)^2}{2}} \leqslant (B(u)+B(v)) - \sqrt{B(u)B(v)}$$
$$\sqrt{B(u)^2+B(v)^2} \leqslant \sqrt{2}[(B(u)+B(v)) - \sqrt{B(u)B(v)}]$$

which implies

$$(B(u)+B(v)) + \sqrt{B(u)^2+B(v)^2} \leqslant (B(u)+B(v)) + \sqrt{2}[(B(u)+B(v)) - \sqrt{B(u)B(v)}]$$
$$\sum_{uv\in E(G)} [(B(u)+B(v)) + \sqrt{B(u)^2+B(v)^2}] \leqslant (1+\sqrt{2}) \sum_{uv\in E(G)} (B(u)+B(v)) - \sqrt{2} \sum_{uv\in E(G)} \sqrt{B(u)B(v)}]$$

Thus

$$KEB(G) \leqslant (1+\sqrt{2})EB_1(G) - \sqrt{2}RPEB(G)$$

□

Theorem 2.3. Let G be a graph. Then

$$KEB(G) < 2EB_1(G)$$

Proof. It is known that for $1 \leqslant x \leqslant y$

$$\sqrt{x^2+y^2}) < (x+y)$$
$$(x+y) + \sqrt{x^2+y^2}) < 2(x+y)$$

setting $x = B(u)$ and $y = B(v)$, we get,

$$B(u)+B(v)) + \sqrt{B(u)^2+B(v)^2} < 2(B(u)+B(v))$$

Thus

$$\sum_{uv\in E(G)} [(B(u)+B(v)) + \sqrt{B(u)^2+B(v)^2}] \leqslant 2 \sum_{uv\in E(G)} (B(u)+B(v))$$

Hence

$$KEB(G) < 2EB_1(G)$$

□

Theorem 2.4. Let G be a graph. Then

$$KEB(G) = EB_1(G) + EBS(G)$$

Proof. We have

$$\sum_{uv \in E(G)} [(B(u) + B(v)) + \sqrt{B(u)^2 + B(v)^2}] = \sum_{uv \in E(G)} (B(u) + B(v)) + \sum_{uv \in E(G)} \sqrt{B(u)^2 + B(v)^2}$$

Hence

$$KEB(G) = EB_1(G) + EBS(G)$$

□

3. RESULTS FOR FRIENDSHIP GRAPHS

A friendship graph F4 is shown in Figure 1. A friendship graph Fn is a graph with 2n+1 vertices and 3n edges. In F_n, there are two types of edges as follows:

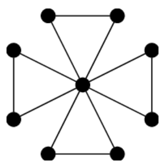

Figure 1: Friendship Graph F4

$E_1 = \{uv \in E(F_n) | d(u) = d(v) = 2\}, \quad |E_1| = n,$
$E_2 = \{uv \in E(F_n) | d(u) = 2, d(v) = 2n\}, \quad |E_2| = 2n.$

Therefore in F_n ,we obtain that $\{B(u), B(v) : uv \in E(F_n)\}$ has two Banhatti edge set partitions.

$BE_1 = \{uv \in E(F_n) | B(u) = B(v) = \frac{2}{2n-1}\}, \quad |BE_1| = n,$
$BE_2 = \{uv \in E(F_n) | B(u) = \frac{2n}{2n-1}, B(v) = 2n\}, \quad |BE_2| = 2n.$

Theorem 3.1. Let Fn be the friendship graph. Then

$$KEB(F_n) = \frac{2n(2+\sqrt{2}) + 2n(4n^2 + 2\sqrt{2}n\sqrt{2n^2 - 2n + 1})}{2n-1}$$

Proof. We have,

$$KEB(F_n) = \sum_{uv\in E(F_n)} [(B(u)+B(v)) + \sqrt{B(u)^2+B(v)^2}]$$

$$= n\left[\frac{2}{2n-1}+\frac{2}{2n-1}+\sqrt{\left(\frac{2}{2n-1}\right)^2+\left(\frac{2}{2n-1}\right)^2}\right] + 2n\left[\frac{2n}{2n-1}+2n+\sqrt{\left(\frac{2n}{2n-1}\right)^2+(2n)^2}\right]$$

After simplification we get ,

$$KEB(F_n) = \frac{2n(2+\sqrt{2}) + 2n(4n^2+2\sqrt{2}n\sqrt{2n^2-2n+1})}{2n-1}$$

□

Theorem 3.2. Let Fn be the friendship graph. Then

$$KEB(F_n, x) = nx^{\left[\frac{2(2+\sqrt{2}}{2n-1}\right]} + 2nx^{\left[\frac{4n^2+2\sqrt{2}n\sqrt{2n^2-2n+1}}{2n-1}\right]}$$

Proof. We have,

$$KEB(F_n, x) = \sum_{uv\in E(F_n)} x^{\left[(B(u)+B(v))+\sqrt{B(u)^2+B(v)^2}\right]}$$

$$= nx^{\left[\frac{2}{2n-1}+\frac{2}{2n-1}+\sqrt{\left(\frac{2}{2n-1}\right)^2+\left(\frac{2}{2n-1}\right)^2}\right]} + 2nx^{\left[\frac{2n}{2n-1}+2n+\sqrt{\left(\frac{2n}{2n-1}\right)^2+(2n)^2}\right]}.$$

After simplification we get ,

$$KEB(F_n, x) = nx^{\left[\frac{2(2+\sqrt{2}}{2n-1}\right]} + 2nx^{\left[\frac{4n^2+2\sqrt{2}n\sqrt{2n^2-2n+1}}{2n-1}\right]}$$

□

Theorem 3.3. Let Fn be the friendship graph. Then

$$m_{KEB(F_n)} = n\left[\frac{2n-1}{2(2+\sqrt{2}}\right] + 2n\left[\frac{2n-1}{4n^2+2\sqrt{2}n\sqrt{2n^2-2n+1}}\right]$$

Proof. We have,

$$m_{KEB(F_n)} = \sum_{uv\in E(F_n)} \frac{1}{\left[(B(u)+B(v))+\sqrt{B(u)^2+B(v)^2}\right]}$$

$$= n\frac{1}{\left[\frac{2}{2n-1}+\frac{2}{2n-1}+\sqrt{\left(\frac{2}{2n-1}\right)^2+\left(\frac{2}{2n-1}\right)^2}\right]} + 2n\frac{1}{\left[\frac{2n}{2n-1}+2n+\sqrt{\left(\frac{2n}{2n-1}\right)^2+(2n)^2}\right]}$$

After simplification we get ,

$$m_{KEB(F_n)} = n\left[\frac{2n-1}{2(2+\sqrt{2}}\right] + 2n\left[\frac{2n-1}{4n^2+2\sqrt{2}n\sqrt{2n^2-2n+1}}\right]$$

□

Theorem 3.4. Let Fn be the friendship graph. Then

$$m_{KEB(F_n,x)} = nx^{\left[\frac{2n-1}{2(2+\sqrt{2}}\right]} + 2nx^{\left[\frac{2n-1}{4n^2+2\sqrt{2}n\sqrt{2n^2-2n+1}}\right]}$$

Proof. We have,

$$m_{KEB(F_n,x)} = \sum_{uv\in E(F_n)} x^{\frac{1}{\left[(B(u)+B(v))+\sqrt{B(u)^2+B(v)^2}\right]}}$$

$$= nx^{\left[\frac{1}{\frac{2}{2n-1}+\frac{2}{2n-1}+\sqrt{\left(\frac{2}{2n-1}\right)^2+\left(\frac{2}{2n-1}\right)^2}}\right]} + 2nx^{\left[\frac{1}{\frac{2n}{2n-1}+2n+\sqrt{\left(\frac{2n}{2n-1}\right)^2+(2n)^2}}\right]}$$

After simplification we get ,

$$m_{KEB(F_n,x)} = nx^{\left[\frac{2n-1}{2(2+\sqrt{2}}\right]} + 2nx^{\left[\frac{2n-1}{4n^2+2\sqrt{2}n\sqrt{2n^2-2n+1}}\right]}$$

□

4. RESULTS FOR WHEEL GRAPHS

A wheel graph W_n is the join of C_n and K_1. Then W_n has n+1 vertices and 2n edges. A graph W_n is presented in Figure 2.

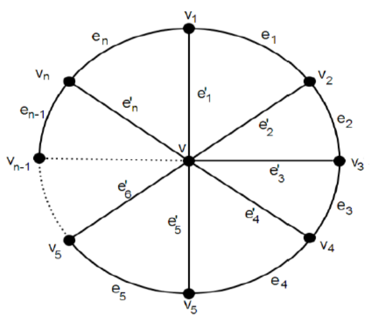

Figure 2: Wheel Graph W_n

In W_n, there are two types of edges as follows:

$E_1 = \{uv \in E(W_n) | d(u) = d(v) = 3\}$, $|E_1| = n$
$E_2 = \{uv \in E(W_n) | d(u) = 3, d(v) = n\}$, $|E_2| = n$

Therefore in W_n ,there are two types of Banhatti edges based on Banhatti degrees of end vertices of each edge follow:

$BE_1 = \{uv \in E(W_n) | B(u) = B(v) = \frac{4}{n-2}\}, \qquad |BE_1| = n.$
$BE_2 = \{uv \in E(W_n) | B(u) = \frac{n+1}{n-2}, B(v) = n+1\}, \qquad |BE_2| = n.$

Theorem 4.1. Let W_n be the Wheel graph. Then

$$KEB(W_n) = \frac{4n(2+\sqrt{2}) + n\left[(n^2-1) + (n^4 - 2n^3 - 2n^2 + 6n + 5)\right]}{n-2}$$

Proof. We have,

$$KEB(W_n) = \sum_{uv \in E(W_n)} [(B(u) + B(v)) + \sqrt{B(u)^2 + B(v)^2}]$$

$$= n\left[\frac{4}{n-2} + \frac{4}{n-2} + \sqrt{\left(\frac{4}{n-2}\right)^2 + \left(\frac{4}{n-2}\right)^2}\right] + n\left[\frac{n+1}{n-2} + (n+1) + \sqrt{\left(\frac{n+1}{n-2}\right)^2 + (n+1)^2}\right]$$

After simplification we get ,

$$KEB(W_n) = \frac{4n(2+\sqrt{2}) + n\left[(n^2-1) + (n^4 - 2n^3 - 2n^2 + 6n + 5)\right]}{n-2}$$

□

Theorem 4.2. Let W_n be the Wheel graph. Then

$$KEB(W_n, x) = nx^{\left[\frac{4(2+\sqrt{2})}{n-2}\right]} + nx^{\left[\frac{\left(n^2-1\right))+\sqrt{n^4-2n^3-2n^2+6n+5}}{n-2}\right]}$$

Proof. We have,

$$KEB(W_n, x) = \sum_{uv \in E(W_n)} x^{\left[(B(u)+B(v))+\sqrt{B(u)^2+B(v)^2}\right]}$$

$$= nx^{\left[\frac{4}{n-2}+\frac{4}{n-2}+\sqrt{\left(\frac{4}{n-2}\right)^2+\left(\frac{4}{n-2}\right)^2}\right]}$$
$$+ nx^{\left[\frac{n+1}{n-2}+(n+1)+\sqrt{\left(\frac{n+1}{n-2}\right)^2+(n+1)^2}\right]}.$$

After simplification we get ,

$$KEB(W_n, x) = nx^{\left[\frac{4(2+\sqrt{2})}{n-2}\right]} + nx^{\left[\frac{\left(n^2-1\right))+\sqrt{n^4-2n^3-2n^2+6n+5}}{n-2}\right]}$$

□

Theorem 4.3. Let W_n be the wheel graph. Then

$$m_{KEB(W_n)} = n\left[\frac{n-2}{4(2+\sqrt{2})}\right] + n\left[\frac{n-2}{(n^2-1)+\sqrt{n^4-2n^3-2n^2+6n+5}}\right]$$

Proof. We have,

$$m_{KEB(W_n)} = \sum_{uv\in E(W_n)} \frac{1}{\left[(B(u)+B(v))+\sqrt{B(u)^2+B(v)^2}\right]}$$

$$= n\frac{1}{\left[\frac{4}{n-2}+\frac{4}{n-2}+\sqrt{\left(\frac{4}{n-2}\right)^2+\left(\frac{4}{n-2}\right)^2}\right]} + n\frac{1}{\left[\frac{n+1}{n-2}+(n+1)+\sqrt{\left(\frac{n+1}{n-2}\right)^2+(n+1)^2}\right]}$$

After simplification we get ,

$$m_{KEB(W_n)} = n\left[\frac{n-2}{4(2+\sqrt{2})}\right] + n\left[\frac{n-2}{(n^2-1)+\sqrt{n^4-2n^3-2n^2+6n+5}}\right]$$

□

Theorem 4.4. Let W_n be the Wheel graph. Then

$$m_{KEB(W_n,x)} = nx^{\left[\frac{n-2}{4(2+\sqrt{2})}\right]} + nx^{\left[\frac{n-2}{(n^2-1)+\sqrt{n^4-2n^3-2n^2+6n+5}}\right]}$$

Proof. We have,

$$m_{KEB(W_n,x)} = \sum_{uv\in E(W_n)} x^{\frac{1}{\left[(B(u)+B(v))+\sqrt{B(u)^2+B(v)^2}\right]}}$$

$$= nx^{\left[\frac{1}{\frac{4}{n-2}+\frac{4}{n-2}+\sqrt{\left(\frac{4}{n-2}\right)^2+\left(\frac{4}{n-2}\right)^2}}\right]} + nx^{\left[\frac{1}{\frac{n+1}{n-2}+(n+1)+\sqrt{\left(\frac{n+1}{n-2}\right)^2+(n+1)^2}}\right]}$$

After simplification we get ,

$$m_{KEB(W_n,x)} = nx^{\left[\frac{n-2}{4(2+\sqrt{2})}\right]} + nx^{\left[\frac{n-2}{(n^2-1)+\sqrt{n^4-2n^3-2n^2+6n+5}}\right]}$$

□

5. RESULTS FOR CHAIN SILICATE NETWORKS

Silicates are very important elements of Earth's crust. Sand and several minerals are constituted by silicates. A family of chain silicate network is symbolized by CS_n and is obtained by arranging n 2 tetrahedral linearly, see Figure 3.

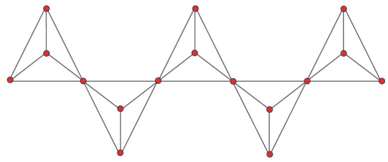

Figure 3: Chain Silicate Network

Let G be the graph of a chain silicate network CS_n with 3n+1 vertices and 6n edges. In G, by calculation, there are three types of edges based on the degree of end vertices of each edge as follows:

$E_1 = \{uv \in E(CS_n) | d(u) = d(v) = 3\}, \qquad |E_1| = n + 4.$
$E_2 = \{uv \in E(CS_n) | d(u) = 3, d(v) = 6\}, \qquad |E_2| = 4n - 2.$
$E_3 = \{uv \in E(CS_n) | d(u) = d(v) = 6\}, \qquad |E_2| = n - 2.$

Therefore in CS_n ,there are three types of Banhatti edges based on Banhatti degrees of end vertices of each edge follow:

$BE_1 = \{uv \in E(W_n) | B(u) = B(v) = \frac{4}{3n-2}\}, \qquad |BE_1| = n + 4.$
$BE_2 = \{uv \in E(W_n) | B(u) = \frac{7}{3n-2}, B(v) = \frac{7}{3n-5}\}, \qquad |BE_2| = 4n - 2.$
$BE_3 = \{uv \in E(W_n) | B(u) = \frac{10}{3n-5}, B(v) = \frac{10}{3n-5}\}, \qquad |BE_2| = n - 2.$

Theorem 5.1. Let CS_n be the Chain Silicate Network . Then

$$KEB(CS_n) = \left(\frac{n+4}{3n-2}\right) 4(2+\sqrt{2}) + (4n-2)\left[\frac{42n-49}{(3n-2)(3n-5} + \sqrt{\frac{882n^2-2058n+1421}{(9n^2-12n+4)(9n^2-30n+25)}}\right] + \left(\frac{n-2}{3n-5}\right) 10\left(2+\sqrt{2}\right)$$

Proof. We have,

$$KEB(CS_n) = \sum_{uv \in E(CS_n)} [(B(u) + B(v)) + \sqrt{B(u)^2 + B(v)^2}]$$

$$\begin{aligned}
&= (n+4)\left[\frac{4}{3n-2} + \frac{4}{3n-2} + \sqrt{\left(\frac{4}{3n-2}\right)^2 + \left(\frac{4}{3n-2}\right)^2}\right] \\
&+ (4n-2)\left[\frac{7}{3n-2} + \frac{7}{3n-5} + \sqrt{\left(\frac{7}{3n-2}\right)^2 + \left(\frac{7}{3n-5}\right)^2}\right] \\
&+ (n-2)\left[\frac{10}{3n-5} + \frac{10}{3n-5} + \sqrt{\left(\frac{10}{3n-5}\right)^2 + \left(\frac{10}{3n-5}\right)^2}\right]
\end{aligned}$$

After simplification we get ,

$$KEB(CS_n) = \left(\frac{n+4}{3n-2}\right) 4(2+\sqrt{2}) + (4n-2)\left[\frac{42n-49}{(3n-2)(3n-5} + \sqrt{\frac{882n^2-2058n+1421}{(9n^2-12n+4)(9n^2-30n+25)}}\right] + \left(\frac{n-2}{3n-5}\right) 10\left(2+\sqrt{2}\right)$$

□

Theorem 5.2. Let CS_n be the Chain Silicate Network. Then

$$KEB(CS_n, x) = (n+4)x^{\left[\frac{4(2+\sqrt{2})}{3n-2}\right]} + (4n-2)x^{\left[\frac{42n-49}{(3n-2)(3n-5} + \sqrt{\frac{882n^2-2058n+1421}{(9n^2-12n+4)(9n^2-30n+25)}}\right]} + (n-2)x^{\left[\frac{10(2+\sqrt{2})}{3n-5}\right]}$$

Proof. We have,

$$KEB(W_n, x) = \sum_{uv\in E(W_n)} x^{\left[(B(u)+B(v))+\sqrt{B(u)^2+B(v)^2}\right]}$$

$$= nx^{\left[\frac{4}{n-2}+\frac{4}{n-2}+\sqrt{\left(\frac{4}{n-2}\right)^2+\left(\frac{4}{n-2}\right)^2}\right]} + nx^{\left[\frac{n+1}{n-2}+(n+1)+\sqrt{\left(\frac{n+1}{n-2}\right)^2+(n+1)^2}\right]}.$$

After simplification we get ,

$$KEB(CS_n, x) = (n+4)x^{\left[\frac{4(2+\sqrt{2})}{3n-2}\right]} + (4n-2)x^{\left[\frac{42n-49}{(3n-2)(3n-5} + \sqrt{\frac{882n^2-2058n+1421}{(9n^2-12n+4)(9n^2-30n+25)}}\right]} + (n-2)x^{\left[\frac{10(2+\sqrt{2})}{3n-5}\right]}$$

□

Theorem 5.3. Let CS_n be the Chain Silicate Network. Then

$$m_{KEB(CS_n)} = (n+4)\left[\frac{3n-2}{4(2+\sqrt{2})}\right] + (4n-2)\left[\frac{1}{\frac{42n-49}{(3n-2)(3n-5)} + \sqrt{\frac{882n^2-2058n+1421}{(9n^2-12n+4)(9n^2-30n+25)}}}\right] + (n-2)\left[\frac{(3n-5)}{10\left(2+\sqrt{2}\right)}\right]$$

Proof. We have,

$$m_{KEB(CS_n)} = \sum_{uv \in E(CS_n)} \frac{1}{\left[(B(u)+B(v)) + \sqrt{B(u)^2+B(v)^2}\right]}$$

$$= (n+4)\frac{1}{\left[\frac{4}{3n-2}+\frac{4}{3n-2}+\sqrt{\left(\frac{4}{3n-2}\right)^2+\left(\frac{4}{3n-2}\right)^2}\right]}$$
$$+(4n-2)\frac{1}{\left[\frac{7}{3n-2}+\frac{7}{3n-5}+\sqrt{\left(\frac{7}{3n-2}\right)^2+\left(\frac{7}{3n-5}\right)^2}\right]}$$
$$+(n-2)\frac{1}{\left[\frac{10}{3n-5}+\frac{10}{3n-5}+\sqrt{\left(\frac{10}{3n-5}\right)^2+\left(\frac{10}{3n-5}\right)^2}\right]}$$

After simplification we get ,

$$m_{KEB(CS_n)} = (n+4)\left[\frac{3n-2}{4(2+\sqrt{2})}\right] + (4n-2)\left[\frac{1}{\frac{42n-49}{(3n-2)(3n-5)}+\sqrt{\frac{882n^2-2058n+1421}{(9n^2-12n+4)(9n^2-30n+25)}}}\right]$$
$$+(n-2)\left[\frac{(3n-5)}{10\left(2+\sqrt{2}\right)}\right]$$

□

Theorem 5.4. Let CS_n be the Chain Silicate Network. Then

$$m_{KEB(CS_n,x)} = (n+4)x^{\left[\frac{3n-2}{4(2+\sqrt{2})}\right]} + (4n-2)x^{\left[\frac{1}{\frac{42n-49}{(3n-2)(3n-5)}+\sqrt{\frac{882n^2-2058n+1421}{(9n^2-12n+4)(9n^2-30n+25)}}}\right]}$$
$$+(n-2)x^{\left[\frac{(3n-5)}{10\left(2+\sqrt{2}\right)}\right]}$$

Proof. We have,

$$m_{KEB(CS_n,x)} = \sum_{uv \in E(CS_n)} x^{\frac{1}{\left[(B(u)+B(v))+\sqrt{B(u)^2+B(v)^2}\right]}}$$

$$= (n+4)x^{\frac{1}{\left[\frac{4}{3n-2}+\frac{4}{3n-2}+\sqrt{\left(\frac{4}{3n-2}\right)^2+\left(\frac{4}{3n-2}\right)^2}\right]}}$$
$$+(4n-2)x^{\frac{1}{\left[\frac{7}{3n-2}+\frac{7}{3n-5}+\sqrt{\left(\frac{7}{3n-2}\right)^2+\left(\frac{7}{3n-5}\right)^2}\right]}}$$
$$+(n-2)x^{\frac{1}{\left[\frac{10}{3n-5}+\frac{10}{3n-5}+\sqrt{\left(\frac{10}{3n-5}\right)^2+\left(\frac{10}{3n-5}\right)^2}\right]}}$$

After simplification we get ,

$$m_{KEB(CS_n,x)} = (n+4)x^{\left[\frac{3n-2}{4(2+\sqrt{2})}\right]} + (4n-2)x^{\left[\frac{1}{\frac{42n-49}{(3n-2)(3n-5)}+\sqrt{\frac{882n^2-2058n+1421}{(9n^2-12n+4)(9n^2-30n+25)}}}\right]}$$
$$+(n-2)x^{\left[\frac{(3n-5)}{10(2+\sqrt{2})}\right]}$$

□

CONCLUSIONS

We have introduced the Kepler E-Banhatti and modified Kepler E-Banhatti indices and their corresponding exponentials of a graph. Furthermore the Kepler E-Banhatti and modified Kepler E-Banhatti indices and their exponentials for friendship graph, wheel graph, chain silicate networks are determined. Also some mathematical properties of Kepler E-Banhatti index are obtained.

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